Updating Boer-Mulders functions from unpolarized pd and pp Drell-Yan data

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We extract the Boer-Mulders functions for the proton by combining the unpolarized pd and pp Drell-Yan data measured by the E866/NuSea Collaboration by the assumption that the $\cos 2\phi$ asymmetry is from the Boer-Mulders function. Using the extracted Boer-Mulders functions, we present the predictions for the $\cos 2\phi$ asymmetries in future pp experiments at J-PARC and $p\bar{p}$ experiments at PANDA and PAX.

I. INTRODUCTION

In a recent work [1] the Boer-Mulders functions for the proton have been extracted from the $\cos 2\phi$ angular asymmetry measured in the unpolarized pd Drell-Yan process by the E866/NuSea Collaboration [2] at FNAL. Much earlier, the first $\cos 2\phi$ asymmetries of dilepton production had been measured two decades ago by the NA10 [3] Collaboration and E165 [4] Collaboration, but for the π-nucleus Drell-Yan processes, showing that the magnitude of the asymmetries is around 30% at most. This substantial $\cos 2\phi$ angular dependence that violates the so-called Lam-Tung relation [5] predicted by perturbative QCD, belongs to the remaining challenges which need to be understood from QCD dynamics. Several attempts have been made to interpret these data, including QCD vacuum effects [6, 7] and higher-twist mechanisms [8, 9]. Furthermore, in the last decade significant efforts have been put forward on the understanding of the $\cos 2\phi$ angular dependence from the view point of the transverse momentum dependent (TMD) Boer-Mulders function $h_1^\perp(x, \boldsymbol{p}_T^2)$ [10]. It was shown [11] by Boer that the angular dependence can be related to the product of two functions $h_1^\perp$, each of which comes from one of the incident hadrons. The Boer-Mulders function describes a correlation between the transverse spin and the transverse momentum of a quark inside an unpolarized hadron. Despite the naive T-odd nature of the Boer-Mulders functions and of their chiral-even partner the Sivers functions [12], it has been shown that they can originate from inital/final state interactions [13, 14] between the struck quark and the spectator of the nucleon, which are important to ensure the gauge invariance of the TMD distribution functions [15–17]. Recently a lot of theoretical studies and phenomenological analysis [18–37] on Boer-Mulders functions have been carried out.

The first measurement of the $\cos 2\phi$ asymmetry in the nucleon-nucleon interacting Drell-Yan process by the E866/NuSea Collaboration makes the attempt on extracting the proton Boer-Mulders function possible. In Ref. [1] we parameterized the Boer-Mulders functions for u, d, $\bar{u}$ and $\bar{d}$ quarks, which were fitted to the E866/NuSea pd data, based on the assumption that the $\cos 2\phi$ asymmetry comes only from the Boer-Mulders effect in the region $q_T^2 \ll Q^2$, where q_T and Q are the transverse momentum and the invariant mass of the lepton pair. In our parametrization the transverse momentum

dependence of $h_1^\perp(x, \boldsymbol{p}_T^2)$ was modeled by the Gaussian ansatz:

$$h_1^{\perp q}(x, \boldsymbol{p}_T^2) = h_1^{\perp q}(x)\, \frac{1}{\pi p_{bm}^2}\, \exp\left(-\frac{\boldsymbol{p}_T^2}{p_{bm}^2}\right). \quad (1)$$

The parameter p_{bm}^2 describes the Gaussian width of the transverse momentum distribution. The x-dependence of $h_1^{\perp q}(x)$ was further parameterized to be proportional to the unpolarized parton distribution $h_1^{\perp q}(x) = x^c (1-x) f_1^q(x)$, where the parameter c is the same for all flavors. The large-x behavior of $h_1^{\perp q}(x)$ compared with $f_1^q(x)$ was taken into account by the factor$(1-x)$ following the argument in Ref. [38].

More recently the E866/NuSea Collaboration reported new measurements [39] on the $\cos 2\phi$ asymmetry in the unpolarized pp Drell-Yan process. The overall magnitude of the $\cos 2\phi$ dependence for pp processes is consistent with, but slightly larger than that of pd processes. The new pp data, besides the previous pd data, will provide further information and constraint on the shape of the Boer-Mulders functions for different flavors. Therefore there is the need to perform a new extraction of Boer-Mulders function in the presence of new pp data. In this work we combine the previous pd data and the new pp data to extract Boer-Mulders functions for the proton to update our previous results. Furthermore, in the new fit we include x_f-dependent and Q-dependent data that have not been applied in the previous extraction. We then apply our extracted Boer-Mulders functions to predict the $\cos 2\phi$ asymmetries in future pp experiments at J-PARC and $p\bar{p}$ experiments at PANDA and PAX.

II. DESCRIPTION OF $\cos 2\phi$ ASYMMETRIES IN TERMS OF BOER-MULDERS FUNCTIONS

The angular differential cross section for unpolarized Drell-Yan processes has the general form

$$\begin{aligned}
\frac{1}{\sigma}\frac{d\sigma}{d\Omega} &= \frac{3}{4\pi}\frac{1}{\lambda+3}(1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi \\
&\quad + \frac{\nu}{2}\sin^2\theta\cos 2\phi).
\end{aligned} \quad (2)$$

where θ and ϕ are, respectively, the polar angle and the azimuthal angle of dileptons in the Collins-Soper frame [40].

The coefficients λ, μ and ν do not depend on these angles, and for scattering that has azimuthal symmetry their values are $\mu = \nu = 0$.

This angular distribution has been measured in muon pair production by pion-nucleon collisions: $\pi^- N \rightarrow \mu^+\mu^- X$, with N denoting a nucleon in deuterium or tungsten, and for a π^- beam with energies of 140, 194, 286 GeV [3] and 252 GeV [4]. The experimental data show large values of ν, near 30%. The most recent measurements of the angular distribution were performed by the E866 Collaboration [2], in pd Drell-Yan processes at 800 GeV/c. The measured ν is about several percent, a result which can not be explained by leading-twist collinear factorization [1] in QCD. As proposed by Boer [11], the non-zero $\cos 2\phi$ term can be produced by the product of two $h_1^\perp$s, each coming from one of the two incident hadrons.

The leading order unpolarized Drell-Yan cross section expressed in the Collins-Soper frame [40] is [11]

$$\frac{d\sigma(h_1 h_2 \rightarrow l\bar{l}X)}{d\Omega dx_1 dx_2 d^2\boldsymbol{q}_T} = \frac{\alpha^2}{3Q^2}\sum_{q,\bar{q}}\left\{A(y)\mathcal{F}[f_1^q f_1^{\bar{q}}] + B(y)\right.$$
$$\left. \times \cos 2\phi \mathcal{F}\left[((2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T) - (\boldsymbol{p}_T\cdot\boldsymbol{k}_T))\frac{h_1^{\perp q} h_1^{\perp \bar{q}}}{M_1 M_2}\right]\right\},$$
$$(3)$$

where $Q^2 = q^2$ and $\boldsymbol{q}_T$ are the invariance mass square and the transverse momentum of the lepton pair. The vector $\hat{\boldsymbol{h}} = \boldsymbol{q}_T/Q_T$. We have used the notation

$$\mathcal{F}[f\bar{f}] = \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T \delta^2(\boldsymbol{p}_T + \boldsymbol{k}_T - \boldsymbol{q}_T)f(x_1, \boldsymbol{p}_T^2)\bar{f}(x_2, \boldsymbol{k}_T^2). (4)$$

The first term in Eq. (3) is azimuthal independent, while the second term has a $\cos 2\phi$ azimuthal dependent term which contributes to the asymmetry ν.

In the case of the pd and pp Drell-Yan processes, the $\cos 2\phi$ asymmetry can be expressed as

$$\nu_{pd}(x_1, x_2, q_T) = \frac{F_{pd}(x_1, x_2, q_T)}{M_p^2 G_{pd}(x_1, x_2, q_T)}, \quad (5)$$

$$\nu_{pp}(x_1, x_2, q_T) = \frac{F_{pp}(x_1, x_2, q_T)}{M_p^2 G_{pp}(x_1, x_2, q_T)}, \quad (6)$$

where

$$F_{pd}(x_1, x_2, q_T) = 2\mathcal{F}\left[(2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T)(e_u^2 h_1^{\perp u}\right.$$
$$\left. + e_d^2 h_1^{\perp d})(h_1^{\perp \bar{u}} + h_1^{\perp \bar{d}})\right] + (q \leftrightarrow \bar{q}), \quad (7)$$

$$G_{pd}(x_1, x_2, q_T) = \mathcal{F}\left[(e_u^2 f_1^u + e_d^2 f_1^d)(f_1^{\bar{u}} + f_1^{\bar{d}})\right] + (q \leftrightarrow \bar{q}), (8)$$

$$F_{pp}(x_1, x_2, q_T) = 2\mathcal{F}\left[(2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T)(e_u^2 h_1^{\perp u} h_1^{\perp \bar{u}}\right.$$

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[1] In has been shown in Ref. [41] that the $\cos 2\phi$ asymmetries can be explained by taking into account the twist-three quark-gluon correlations in collinear factorization which is consistent with the Boer-Mulders effect in the TMD factorization approach.
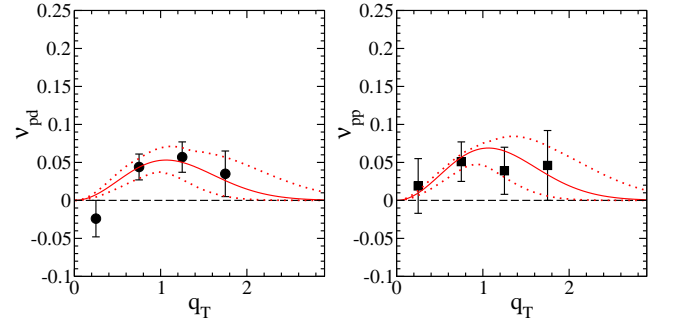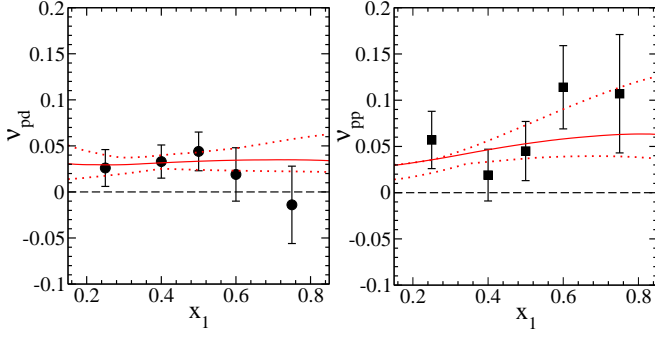

FIG. 1: The q_T-dependent $\cos 2\phi$ asymmetries for unpolarized pd (left) and pp (right) Drell-Yan process calculated from our fitted results. The region between the two dotted line corresponds to the uncertainty of the parameters. Data are from the FNAL E866/NuSea experiments. Here we exclude data with $p_T > 2$ GeV.

$$\left. + e_d^2 h_1^{\perp d} h_1^{\perp \bar{d}})\right] + (q \leftrightarrow \bar{q}), \quad (9)$$

$$G_{pp}(x_1, x_2, q_T) = \mathcal{F}[(e_u^2 f_1^u f_1^{\bar{u}} + e_d^2 f_1^d f_1^{\bar{d}})] + (q \leftrightarrow \bar{q}). \quad (10)$$

For distribution functions for deuteron, we have used the isospin relation:

$$f^{u/deuteron} \approx f^{u/p} + f^{u/n} = f^u + f^d. \quad (11)$$

In Ref. [1] we have parameterized the transverse momentum dependence of Boer-Mulders functions with a Gaussian form as follows

$$h_1^{\perp q}(x, \boldsymbol{p}_T^2) = h_1^{\perp q}(x)\frac{\exp(-\boldsymbol{p}_T^2/p_{bm}^2)}{\pi p_{bm}^2}. \quad (12)$$

The x dependence for u, d, $\bar{u}$ and $\bar{d}$ quarks is parameterized, as follows

$$h_1^{\perp q}(x) = H_q x^c (1 - x) f_1^q(x). \quad (13)$$

The above parametrizations, with 6 parameters, have been applied to fit pd Drell-Yan data measured by E866/NuSea Collaboration. In the fit the P_T-dependent and $x_{1/2}$-dependent $\cos 2\phi$ asymmetry data were used. The fitted result was employed to predict the x_f-dependent and Q-dependent $\cos 2\phi$ asymmetries which were compared with the corresponding data.

Recently the E866/NuSea Collaboration reports measurements [39] of the $\cos 2\phi$ asymmetries on unpolarized pp Drell-Yan processes at $E_p = 800$ GeV. The new pp data, together with the previous pd data, will provide further information on the shape of the Boer-Mulders functions for different flavors. In this paper we will combine the previous pd data and the new pp data in the fit. Further more we will include x_f-dependent and Q-dependent data in our fit. To do this we will parameterize the Boer-Mulders functions as in Eqs. (12) and (13), but changing the form slightly.

In our previous fit we modeled the x-dependent behavior of $h_1^{\perp q}(x, \boldsymbol{p}_T^2)$ at small x as x^c compared with $f_1^q(x)$, and we assumed the value of c to be flavor independent, as shown in

FIG. 2: The x_1-dependent $\cos 2\phi$ asymmetries for unpolarized pd (left) and pp (right) Drell-Yan process calculated from our fitted results. The region between the two dotted line corresponds to the uncertainty of the parameters. Data are from the FNAL E866/NuSea collaboration.
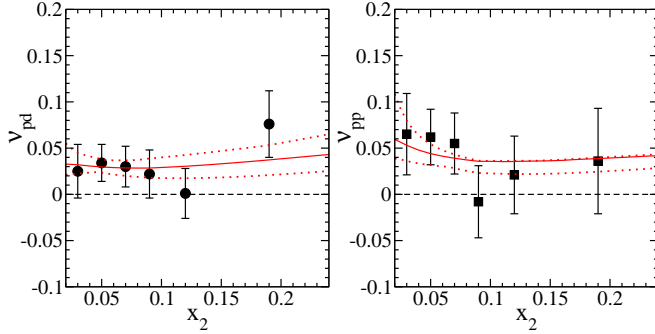

FIG. 4: The x_F-dependent $\cos 2\phi$ asymmetries for unpolarized pd (left) and pp (right) Drell-Yan process calculated from our fitted results. The region between the two dotted line corresponds to the uncertainty of the parameters. Data are from the FNAL E866/NuSea collaboration.

FIG. 3: The x_2-dependent $\cos 2\phi$ asymmetries for unpolarized pd (left) and pp (right) Drell-Yan process calculated from our fitted results. The region between the two dotted line corresponds to the uncertainty of the parameters. Data are from the FNAL E866/NuSea collaboration.

FIG. 5: The Q-dependent $\cos 2\phi$ asymmetries for unpolarized pd (left) and pp (right) Drell-Yan process calculated from our fitted results. The region between the two dotted line corresponds to the uncertainty of the parameters. Data are from the FNAL E866/NuSea collaboration.

(13). Now with more data available, we are able to release this constraint to replace c as c_q, depending on flavor. Secondly we model the large x-dependence of the Boer-Mulders functions by $(1-x)^b$, different from our previous fit in which the large x dependence is $1-x$. Therefore we have the new parametrizations for $h_1^{\perp q}$ ($q = u, d, \bar{u}$ and $\bar{d}$) as follows:

$$h_1^{\perp q}(x) \;=\; H_q\, x^{c^q}\, (1-x)^b\, f_1^q(x), \qquad (14)$$

The first $\boldsymbol{p}_T^2$-moment of Boer-Mulders function is defined as

$$h_1^{\perp (1) q}(x) = \int d^2\boldsymbol{p}_T\, \frac{\boldsymbol{p}_T^2}{2M^2}\, h_1^\perp(x, \boldsymbol{p}_T^2) \qquad (15)$$

From Eqs. (12) and (14) one can calculate $h_1^{\perp (1) q}(x)$ from our parametrization as

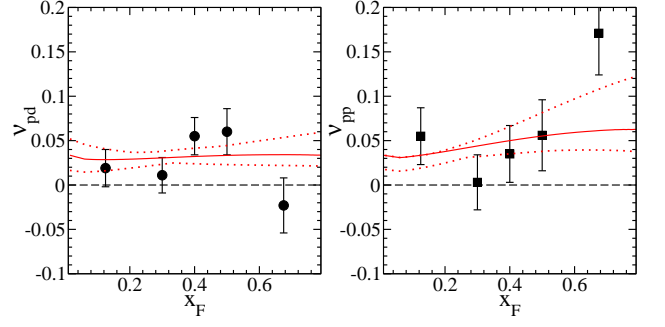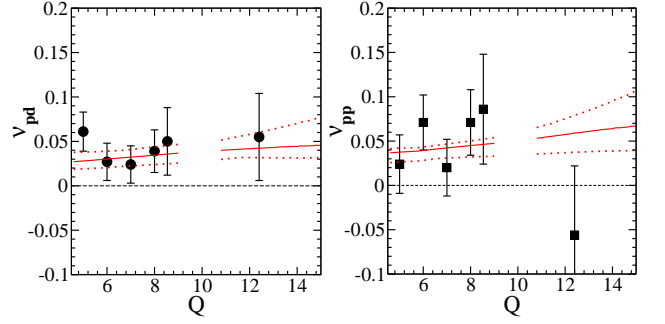$$h_1^{\perp (1) q}(x) = \frac{p_{bm}^2}{2M^2} h_1^{\perp q}(x) \qquad (16)$$

With the Gaussian form for the $\boldsymbol{p}_T$ dependence of Boer-Mulders functions and the unpolarized TMD distribution

$$f_1^q(x, \boldsymbol{p}_T^2) = f_1^q(x)\frac{1}{\pi p_{un}^2} \exp\left(\frac{\boldsymbol{p}_T^2}{p_{un}^2}\right), \qquad (17)$$

the transverse momentum integrations in Eqs. (7) – (10) can be deconvoluted and the results are:

$$F_{pd}(x_1, x_2, q_T) \;=\; F_{pd}(x_1, x_2)\frac{q_T^2}{36\pi p_{bm}^2} \exp\left(-\frac{q_T^2}{2p_{bm}^2}\right), (18)$$

$$F_{pp}(x_1, x_2, q_T) \;=\; F_{pp}(x_1, x_2)\frac{q_T^2}{36\pi p_{bm}^2} \exp\left(-\frac{q_T^2}{2p_{bm}^2}\right), (19)$$

$$G_{pd}(x_1, x_2, q_T) \;=\; G_{pd}(x_1, x_2)\frac{1}{18\pi p_{un}^2} \exp\left(-\frac{q_T^2}{2p_{un}^2}\right), (20)$$

$$G_{pp}(x_1, x_2, q_T) \;=\; G_{pp}(x_1, x_2)\frac{1}{18\pi p_{un}^2} \exp\left(-\frac{q_T^2}{2p_{un}^2}\right). (21)$$

where the $x_{1/2}$ dependent parts are

$$F_{pd}(x_1, x_2) = \left(4\, h_1^{\perp u}(x_1) + h_1^{\perp d}(x_1) \right)$$

$$\times \left(h_1^{\perp \bar{u}}(x_2) + h_1^{\perp \bar{d}}(x_2) \right) + (q \to \bar{q})$$

$$= (1 - x_1)^b (1 - x_2)^b (4 H_1 x_1^{c_u} f_1^u(x_1) x_2^{c_{\bar{u}}} f_1^{\bar{u}}(x_2)$$

$$+ H_2 x_1^{c_d} f_1^d(x_1) x_2^{c_{\bar{d}}} f_1^{\bar{d}}(x_2) + 4 H_3 x_1^{c_u} f_1^u(x_1) x_2^{c_{\bar{d}}} f_1^{\bar{d}}(x_2)$$

$$+ (H_1 H_2 / H_3) x_1^{c_d} f_1^d(x_1) x_2^{c_{\bar{u}}} f_1^{\bar{u}}(x_2)) + (q \to \bar{q}), \qquad (22)$$

$$F_{pp}(x_1, x_2) = 4 h_1^{\perp u}(x_1) h_1^{\perp \bar{u}}(x_2) + h_1^{\perp d}(x_1) h_1^{\perp \bar{d}}(x_2) + (q \to \bar{q})$$

$$= (1 - x_1)^b (1 - x_2)^b (4 H_1 x_1^{c_u} f_1^u(x_1) x_2^{c_{\bar{u}}} f_1^{\bar{u}}(x_2)$$

$$+ H_2 x_1^{c_d} f_1^d(x_1) x_2^{c_{\bar{d}}} f_2^{\bar{d}}(x_2)) + (q \to \bar{q}), \qquad (23)$$

$$G_{pd}(x_1, x_2) = \left(4 f_1^u(x_1) + f_1^d(x_1) \right)$$

$$\times \left(f_1^{\bar{u}}(x_2) + f_1^{\bar{d}}(x_2) \right) + (q \to \bar{q}), \qquad (24)$$

$$G_{pp}(x_1, x_2) = 4 f_1^u(x_1) f_1^{\bar{u}}(x_2) + f_1^d(x_1) f_1^{\bar{d}}(x_2) + (q \to \bar{q}), \qquad (25)$$

where $H_1 = H_u H_{\bar{u}}$, $H_2 = H_d H_{\bar{d}}$, $H_3 = H_u H_{\bar{d}}$ and $H_1 H_2 / H_3 = H_{\bar{u}} H_d$. Since H_u, H_d, $H_{\bar{u}}$ and $H_{\bar{u}}$ always appear as products of two of them, we will apply H_1, H_2 and H_3 as the parameters in the fit. Therefore the actual number of free parameters is reduced to 9.

The q_T-, x_1- and x_2-dependent $\cos 2\phi$ asymmetries in unpolarized pd and pp Drell-Yan processes can then be expressed as

$$\nu_{NN}(q_T) = \frac{\int dx_1 \int dx_2 \, F_{NN}(x_1, x_2, q_T)}{M_p^2 \int dx_1 \int dx_2 \, G_{NN}(x_1, x_2, q_T)}, \qquad (26)$$

$$\nu_{NN}(x_1) = \frac{\int dx_2 \int dq_T^2 \, F_{NN}(x_1, x_2, q_T)}{M_p^2 \int dx_2 \int dq_T^2 \, G_{NN}(x_1, x_2, q_T)}, \qquad (27)$$

$$\nu_{NN}(x_2) = \frac{\int dx_1 \int dq_T^2 \, F_{NN}(x_1, x_2, q_T)}{M_p^2 \int dx_1 \int dq_T^2 \, G_{NN}(x_1, x_2, q_T)}, \qquad (28)$$

where the subscript NN denotes pd and pp.

One can also express the cross-section of the Drell-Yan process, depending on Feynman x_F and the mass of the lepton pair Q as

$$\frac{d\sigma}{dx_F dQ^2 d^2 \boldsymbol{q}_T} = \frac{1}{s \sqrt{x_F^2 + Q^2/s}} \frac{d\sigma}{dx_1 dx_2 d^2 \boldsymbol{q}_T}, \qquad (29)$$

with

$$x_{1/2} = \frac{\pm x_F + \sqrt{x_F^2 + Q^2/s}}{2}, \quad Q^2 = x_1 x_2 s. \qquad (30)$$

Therefore the x_F- and Q-dependent $\cos 2\phi$ asymmetries can be expressed as

$$\nu_{NN}(x_F) = \frac{\int dQ^2 \int dq_T^2 \frac{F_{NN}(x_1, x_2, q_T)}{\sqrt{x_F^2 + Q^2/s}}}{M_p^2 \int dQ^2 \int dq_T^2 \frac{G_{NN}(x_1, x_2, q_T)}{\sqrt{x_F^2 + Q^2/s}}}, \qquad (31)$$

$$\nu_{NN}(Q) = \frac{\int dx_F \int dq_T^2 \frac{F_{pd}(x_1, x_2, q_T)}{\sqrt{x_F^2 + Q^2/s}}}{M_p^2 \int x_F \int q_T^2 \frac{G_{NN}(x_1, x_2, q_T)}{\sqrt{x_F^2 + Q^2/s}}}. \qquad (32)$$
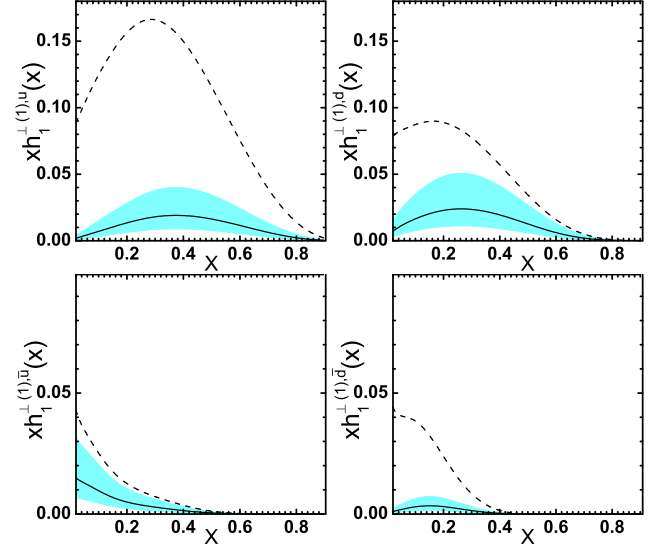

FIG. 6: The first $\boldsymbol{p}_T^2$-moments of Boer-Mulders functions for u, d, $\bar{u}$ and $\bar{d}$ quarks for $Q^2 = 1$ GeV^2 by solid lines, the shadows depict the variation ranges of $x h_1^{\perp (1) q}(x)$ allowed by the positivity bound. The dashed lines show $\frac{\langle p_T \rangle_{un}}{2M} x f_1^q(x)$. .

III. FITTING BOER-MULDERS FUNCTIONS TO THE UNPOLARIZED E866/NUSEA pd AND pp DATA

The E866/NuSea Collaboration measured the $\cos 2\phi$ asymmetryies ν_{pd} and ν_{pd} vs Q_T, x_1, x_2, x_F and $m_{\mu\mu}$ in the following kinematical region:

$$4.5 \,\text{GeV} < Q < 9 \,\text{GeV} \ \text{ and } \ 10.7 \,\text{GeV} < Q < 15 \,\text{GeV},$$
$$q_T < 4 \,\text{GeV}, \quad 0.15 < x_1 < 0.85, \quad 0.02 < x_2 < 0.24.$$

In the following we apply the theoretical expressions (26) - (28), (31) and (32) to fit the unpolarized pd and pp Drell-Yan $\cos 2\phi$ asymmetry data [2, 39]. The Boer-Mulders effect to the $\cos 2\phi$ asymmetry is supposed to apply in the region where q_T is not large. At large q_T, the higher order perturbative QCD contributions [42, 43] might be important. Therefore we exclude the data with $q_T > 2$ GeV in our fit. For the parton distribution $f_1^q(x)$ we adopt the MSTW2008 LO set [44]. We choose the Gaussian width for $f_1^q(x, \boldsymbol{p}_T^2)$ as $p_{un}^2 = 0.25$ GeV^2, following the value given in Refs. [45, 46]. The best fit results and the errors for the parameters are as follows:

$$H_1 = 0.62^{+0.52}_{-0.29}, \quad H_2 = 1.45^{+1.30}_{-1.12}, \quad H_3 = 0.61^{+0.50}_{-0.55},$$
$$c_u = 0.63^{+0.53}_{-0.21}, \quad c_d = 0.47^{+0.36}_{-0.39}, \quad c_{\bar{u}} = 0.07^{+0.06}_{-0.05}, \quad (33)$$
$$c_{\bar{d}} = 0.75^{+0.72}_{-0.52}, \quad b_0 = 0.17^{+0.15}_{-0.14}, \quad p_{bm}^2 = 0.173^{+0.027}_{-0.033}.$$

The χ^2 of this fit is 35.95 for 52 data points, resulting $\chi^2/d.o.f = 0.84$. In Figs. 1, 2, 3, 4 and 5 we show the q_T-, x_1-, x_2-, x_F- and Q-dependent $\cos 2\phi$ asymmetries for unpolarized pd and pp Drell-Yan process calculated from our fitted results and compare them with FNAL E866/NuSea data. The solid lines show the best fit results, and the regions between
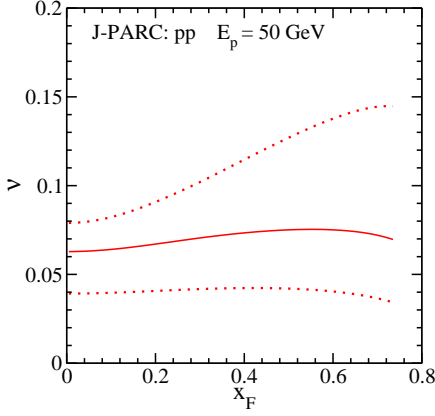
FIG. 7: The $\cos 2\phi$ asymmetries vs x_F at J-PARC with proton beam energy $E_p = 50$ GeV. The region between the two dotted line corresponds to the uncertainty of the parameters.

the two dotted lines correspond to the uncertainty of the parameter. In Fig. 1 we also show the predictions for $p_T > 2GeV$ region from the Boer-Mulders effect.

The possible range of coefficients H_q are obtained from the values of H_1, H_2 and H_3, by employing the positivity bound [47] for $h_1^{\perp q}(x, \boldsymbol{p}_T^2)$ for the entire x and $\boldsymbol{p}_T$ regions:

$$\frac{|p_T h_1^{\perp q}(x, \boldsymbol{p}_T^2)|}{M} \leq f_1^q(x, \boldsymbol{p}_T^2). \tag{34}$$

We have

$$
\begin{aligned}
H_u &= 0.59^{+0.64}_{-0.31}, & H_d &= 1.37^{+1.53}_{-0.72}, \\
H_{\bar{u}} &= 1.10^{+1.21}_{-0.57}, & H_{\bar{d}} &= 1.08^{+1.18}_{-0.56}.
\end{aligned}
\tag{35}
$$

The upper and lower limits for H_q are determined by the positivity bound for $h_1^{\perp q}(x, \boldsymbol{p}_T^2)$. The central value for H_q shown above is obtained from the geometric mean values of the upper and lower limits for H_q: $H_q^{\text{cen}} = \sqrt{H_q^{\max} H_q^{\min}}$. In our previous work [1] the variation range of H_q allowed by the positivity bound was described by the coefficient ω, namely, that the substitution $H_q \to \omega H_q$ for $q = u, d$ and $H_q \to \frac{1}{\omega} H_q$ for $q = \bar{u}, \bar{d}$ will not change the result. In our new fit presented here, the range of ω is $0.48 < \omega < 2.1$, and central values for H_q correspond to $\omega = 1$.

The positivity bound given in (34) implies

$$h_1^{\perp(1)q}(x) \leq \frac{\langle p_T \rangle_{un}}{2M} f_1^q(x). \tag{36}$$

In Fig. 6 we show the first $\boldsymbol{p}_T^2$-moments of the Boer-Mulders functions $x h_1^{\perp(1)q}(x)$ for u, d, $\bar{u}$ and $\bar{d}$ quarks for $Q^2 = 1$ GeV^2 by solid lines, the shadows depict the variation ranges of $x h_1^{\perp(1)q}(x)$ allowed by the positivity bound. The dashed lines show $\frac{\langle p_T \rangle_{un}}{2M} x f_1^q(x)$.

Several comments on our fit are in order. First, since we don't know the scale dependence of Boer-Mulders functions,

we assumed that it has the same behavior as that of the unpolarized distribution f_1^q. Recently there has been growing interest on performing next-to-leading order analysis [48–52] of single-spin asymmetries in Semi-inclusive deeply inelastic scattering and Drell-Yan processes, especially on the Sivers asymmetry. Those studies can provide information on the evolution of the Sivers function. It is also interesting to study the evolution of the Boer-Mulders function. However, to investigate the scale dependence behavior of the Boer-Mulders function and its impacts on the $\cos 2\phi$ asymmetries is out of the scope of this paper. Secondly, we considered the Boer-Mulders effect as the dominant source for the $\cos 2\phi$ asymmetries, which is reasonable in the region $q_T^2 \ll Q^2$. Since the data in E866/NuSea covers the kinematics regime $4.5\,\text{GeV} < Q < 9\,\text{GeV}$ and $10.7\,\text{GeV} < Q < 15\,\text{GeV}$, one expects that the Boer-Mulders effect dominates in the region $q_T < 2$ GeV. At $q_T > 3$ GeV, the higher order perturbative QCD contributions [42, 43] might be important. This can be seen from Fig. 1, which indicates that the predicted asymmetries at large q_T are small, while the size of the data in that region is substantial [2, 39]. Thirdly, the $\chi^2/d.o.f$ in the fit presented in this work is a little bigger than that in the previous fit shown in Ref. [1]. This is because in Ref. [1] we only included 16 data points from the pd process, while now we have 52 data points in our new fit. And among those data we include Q- and x_F-dependent data also. We have checked that the two data points in the large x_F region give rise to substantial contributions (20%) to the total χ^2 in our best fit.

IV. PREDICTIONS FOR FUTURE pp AND $p\bar{p}$ EXPERIMENTS

We will then apply our extracted Boer-Mulders in previous section to predict the $\cos 2\phi$ asymmetries in future pp and $p\bar{p}$ experiments.

Drell-Yan process has been proposed at J-PARC [53] by pp scattering with proton beam energy $E_p = 50$ GeV. with this lower beam energy than that at E866/NuSea, the measurement of $\cos 2\phi$ asymmetry at J-PARC will provide complementary information on Boer-Mulders functions in a different kinematical region. We estimate the x_F- dependent $\cos 2\phi$ asymmetry at J-PARC by imposing the cuts $0 \leq q_T \leq 1$ GeV and $4 \leq Q \leq 5$ GeV from the fitted results in Eq. (33) directly, as shown in Fig. 7. The solid curve shows the result from the best fitted values, while the region between the dotted lines correspond to the parameters uncertainty.

The GSI-PANDA experiment [54] will run the Drell-Yan process with an unpolarized antiproton beam colliding with an unpolarized proton target at $s = 30$ GeV^2. The PAX [55] experiment might also perform unpolarized $p\bar{p}$ Drell-Yan process at the energy $s = 45$ GeV^2 in the fixed target mode. The kinematical cuts we apply in the estimation of the asymmetry for those experiments are $2 \leq Q \leq 3$ GeV and $0 \leq q_T \leq 0.4$ GeV. To estimate the asymmetries in $p\bar{p}$ processes we need to use the H_q results given in (35), rather than to the results for H_i $(i = 1, 2, 3)$ in (33). In Fig. 8 we show the predicted $\cos 2\phi$ asymmetries ν at PANDA for $s = 30$ GeV^2, and PAX
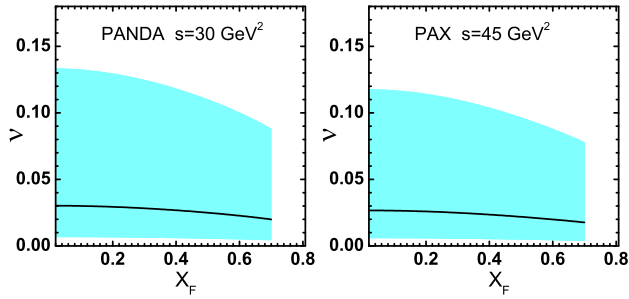
FIG. 8: The $\cos 2\phi$ asymmetries vs x_F at PANDA for $s = 30$ and at PAX for $s = 45$ GeV^2, respectively. The bands correspond to all possible values of ω in the allowed range $0.48 < \omega < 2.1$.

for $s = 45$ GeV^2, as a function of x_F. The solid lines are calculated from the central value of H_q given in Eq. (35), and the bands correspond to all possible values of ω in the allowed range $0.48 < \omega < 2.1$. The asymmetry calculated from the central value of H_q in $p\bar{p}$ process is smaller than that in pp case. Therefore the measurement on the $\cos 2\phi$ asymmetry at PANDA and PAX might provide valuable examination on our extraction of Boer-Mulders functions.

V. SUMMARY

We have parameterized the Boer-Mulders functions for the proton by employing a Gaussian form for their transverse momentum dependence. We then fitted our parameterizations to both the previous measured pd data and recent pp data on the unpolarized Drell-Yan $\cos 2\phi$ dilepton asymmetries from the E866/NuSea Collaboration. The basic assumption in the fit is that the $\cos 2\phi$ asymmetry is contributed by the product of two Boer-Mulders functions and that other contributions such as perturbative QCD effects can be ignored in the region $q_T^2 \ll Q^2$. In our fit we included not only the q_T-, x_1- and x_2-dependent asymmetry data, but also Q- and x_F-dependent data. We applied our extracted Boer-Mulders to predict the $\cos 2\phi$ asymmetries in future pp experiment at J-PARC and $p\bar{p}$ experiment at PANDA and PAX. We found a smaller asymmetry for $p\bar{p}$ processes compared with that for pp processes, which might serve as a test on our extraction of Boer-Mulders functions.

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